An Odd Approximation of Inflation for State Dependent Pricing Models

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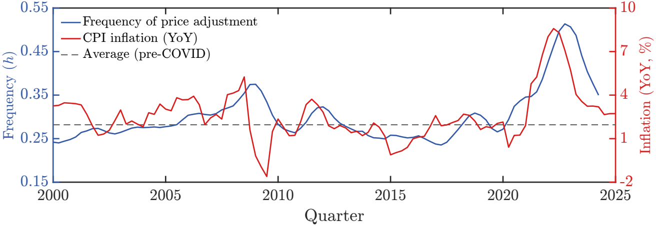
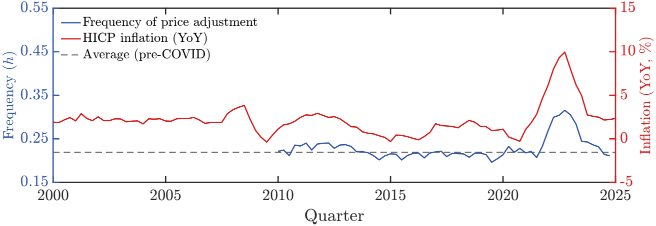
Inflation vs quarterly price adjustment frequency

(a) United States



(b) Euro area



Frequency: Cotton and Garga (2026) for the US, Gautier et al. (2025) for the Euro area. Smoothed with a 3-quarter centered MA.

Motivation

In previous work, we used firm-level price and cost data to show:

- State-dependent pricing model captures inflation both pre- & post-pandemic (GGLT '25)
- Issues:
  - No closed-form solutions; computationally intensive.
  - Firms need to forecast the entire distribution to set prices. (Moll '26)

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This paper: Algebraic approach to state-dependent pricing models

- Characterization of the entire impulse response function of inflation:
  - Cubic (odd) polynomial of the mean of the distribution of price gaps
  - Tractable and simple computationally (Dynare)
  - Provides a quasi-aggregation result
  - Can be combined in DSGE models

Plan of the talk

1. Theoretical framework: State-dependent pricing model with strategic complementarities.
2. Characterization of the IRF of inflation.
3. Quantitative exercises.

Theoretical Framework

Model summary

- Continuum of monopolistically competitive, heterogeneous firms.
- Cobb-Douglas aggregate price index. (Kimball in paper)
- Production function with decreasing RTS. (Mean-field game as in Alvarez et al. 2023)
- Quadratic profit function. (Alvarez et al. 2022)
- Random menu costs. (Caballero Engel 2007)
- Random free price adjustments. ("Calvo Plus" as in Nakamura Steinsson 2010)
- No trend inflation. (relaxed in paper)
- New Keynesian demand block. (later)

A canonical menu cost model with strategic complementarities

- **Nominal marginal cost** (aggregate ξ_t; idiosyncratic $a_t(f)$; decreasing RTS ν)

$$mc_t^n(f) = \xi_t + a_t(f) + \nu\, y_t(f), \qquad a_t(f) = a_{t-1}(f) + \varepsilon_t(f), \qquad \varepsilon_t(f) \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

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- **Static target price** (absent nominal rigidities; strategic complementarities Ω)

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- **Value of the firm** (quadratic profits; $\mathbb{I}_t(f) = 1$ if adjusts; menu cost $\chi_t(f) \sim G(\cdot)$)

$$V_0(f) = \max_{\{p_t(f), \mathbb{I}_t(f)\}_{t\geq 0}} \mathbb{E}_0 \sum_{t\geq 0} \beta^t \Big\{ -B\big(p_t^o(f) - p_t(f)\big)^2 - \chi_t(f)\, \mathbb{I}_t(f) \Big\}$$

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- **Optimal reset price (intensive margin)** (FOC for $p_t(f)$)

$$p_t^\star(f) = \frac{\mathbb{E}_t \sum_{i\geq 0} \beta^i \big(\prod_{j=1}^i (1 - \mathbb{I}_{t+j}^\star(f))\big)\, p_{t+i}^o(f)}{\mathbb{E}_t \sum_{i\geq 0} \beta^i \big(\prod_{j=1}^i (1 - \mathbb{I}_{t+j}^\star(f))\big)}$$

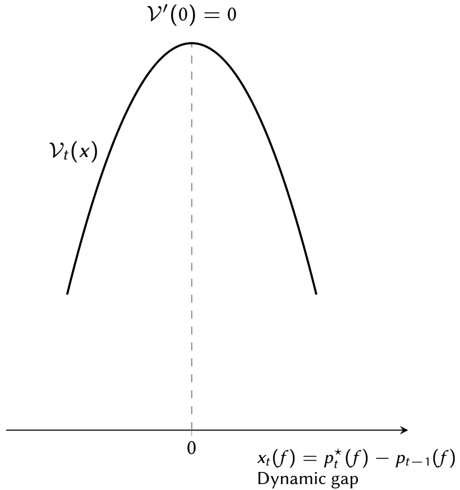
The extensive margin: generalized hazard function

Dynamic price gap (New state variable)

$$x_t(f) := p_t^\star(f) - p_{t-1}(f)$$

Indirect value of not adjusting

$$\mathcal{V}_t(x) = -B\big(\underbrace{p_t^o(f) - p_t^\star(f)}_{x_t^\star} + x\big)^2 + \beta\,\mathbb{E}_t\big[V_{t+1}(f) \mid x_t(f) = x\big]$$



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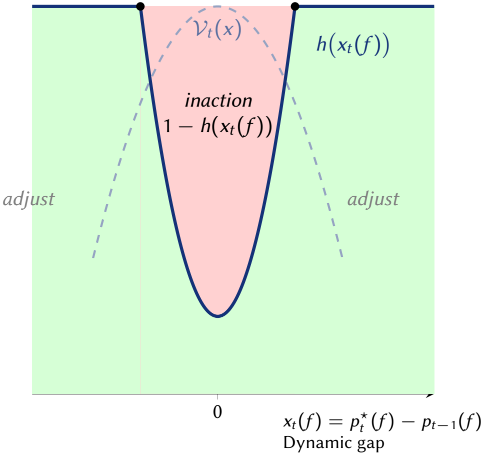
Generalized hazard function

$$h_t(x) := \mathbb{E}_t\big(\mathbb{I}_t(f) \,\big|\, x_t(f) = x\big)$$



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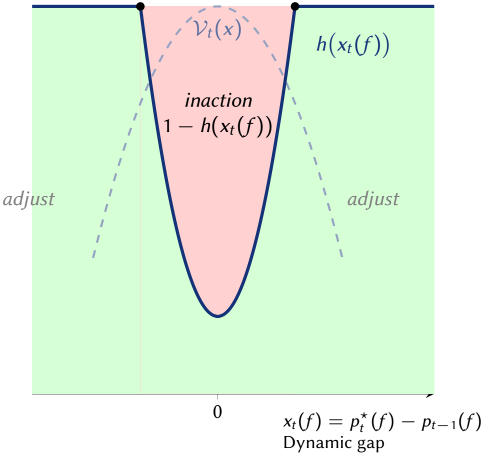
$$h_t(x) = h_t(0) + \big(1 - h_t(0)\big)\,G\Big(\,\mathcal{V}_t(0) - \mathcal{V}_t(x)\,\Big)$$



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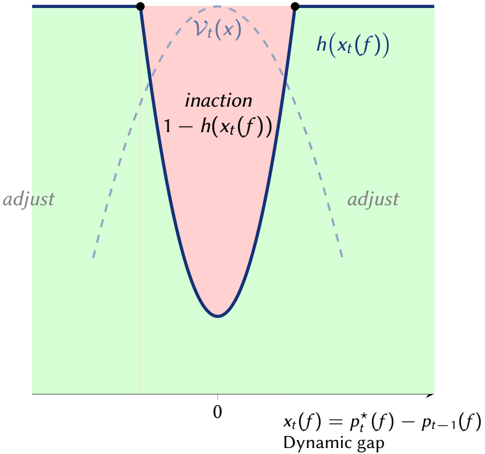
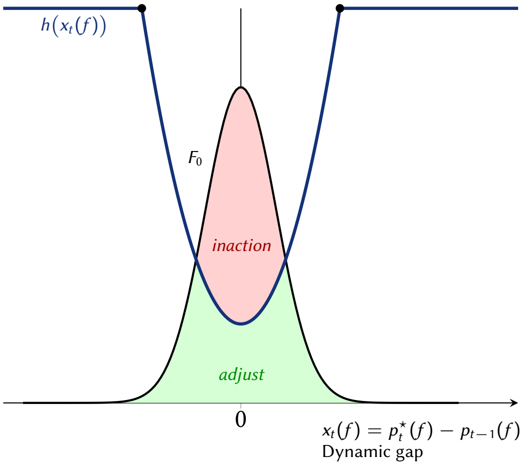
Quadratic approximation of the GHF

$$h_t\big(x_t(f)\big) = h_t(0) + \phi_t\, x_t^2(f) + o\big(x_t^2(f)\big)$$



The effect of an aggregate shock



Dynamic gap distribution

$$F_0(x) := \Pr\left(x_0(f) \leq x\right)$$

Average price gap

$$x_0 := \int x \, dF_0(x)$$

Cross-sectional variance

$$\sigma_0^2 := \int x^2 \, dF_0(x) - x_0^2$$

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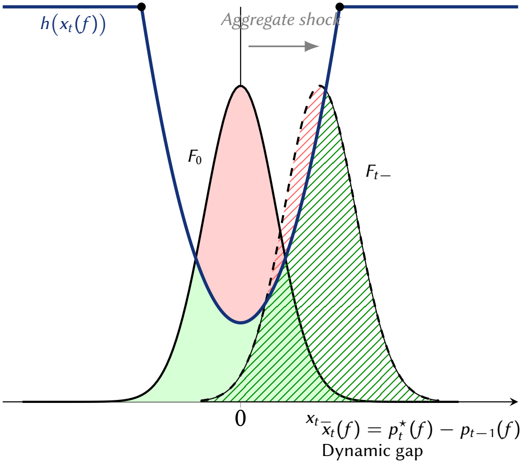

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Cross-sectional variance

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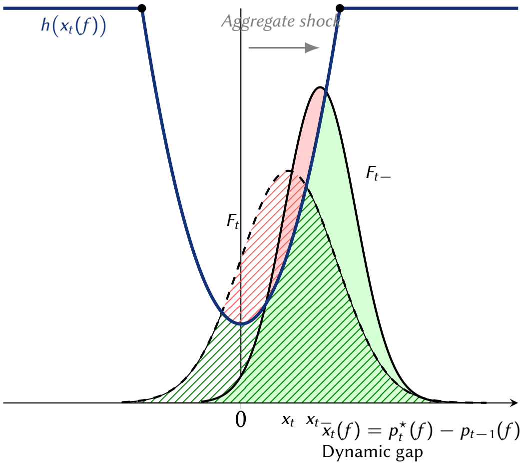

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Odd (cubic) approximation of inflation

Inflation

$$\pi_t = \int h\big(x_t(f)\big)\, x_t(f)\, dF_t$$

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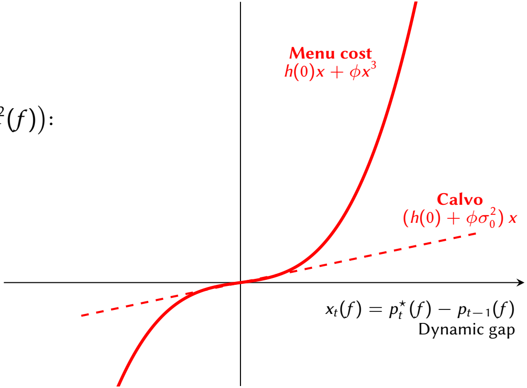

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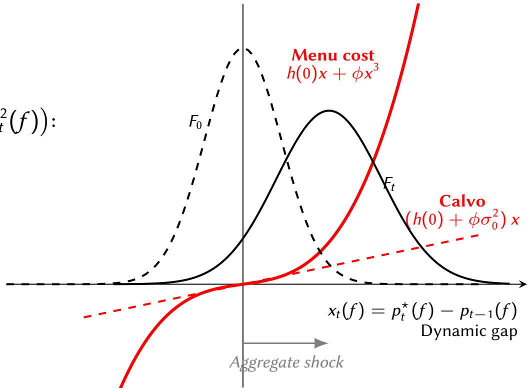

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(Step 3) Approximation of variance:

$$\sigma_t^2 = \sigma_0^2 + \sigma_{xx}^2\, x_t^2 + o(x_t^2)$$

$$\boxed{\Rightarrow\ \pi_t = \left(h_0 + 3\phi\sigma_0^2\right) x_t + \phi\left(1 + 3\sigma_{xx}^2\right) x_t^3 + o\left(x_t^3\right)}$$

Variance derivations

The impulse response functions of inflation and frequency

Theorem. *Assume stationary environment. Consider an aggregate shock of size δ, with average price-gap response $x(\delta)$. The impulse responses of the frequency of price adjustment and of inflation are, to second and third order respectively:*

$$h(\delta) = \alpha_0 + \alpha_2\, x^2(\delta) + o(\delta^2)$$
$$\pi(\delta) = \beta_1\, x(\delta) + \beta_3\, x^3(\delta) + o(\delta^3)$$

where $\{\alpha_0, \alpha_2, \beta_1, \beta_3\}$ are functions of the parameters of the steady state $\{h(0), \phi, \sigma_\varepsilon^2\}$ only.

closed-form coefficients

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closed-form coefficients

Inflation decomposition

$$\pi(\delta) \simeq \underbrace{\alpha_0\, x(\delta)}_{\text{Calvo}} + \underbrace{\alpha_2\, x^3(\delta)}_{\text{Frequency}} + \underbrace{(\beta_1 - \alpha_0)\, x(\delta) + (\beta_3 - \alpha_2)\, x^3(\delta)}_{\text{Selection}}$$

Solving the global model: computational challenges

The global solution must track the **entire price-gap distribution** F_t:

1. **Infinite-dimensional state** — law of motion of F_t is nonlinear (selection + drift + diffusion); each moment depends on higher moments.
2. **Mean-field fixed point** — reset prices $p_t^\star(f)$ depend on the *future path* of F_t, itself generated by firms' collective policies.
3. **General-equilibrium nesting** — the aggregate drift (nominal marginal cost) is itself an equilibrium object $\leftarrow$ output, wages, price level.

$\Rightarrow$ global methods are costly and *preclude* estimation, filtering, and counterfactuals.

This paper: collapse F_t to the single scalar $x_t \;\Rightarrow\;$ standard perturbation (Dynare).

Extensions: trend inflation (asymmetric responses), skewed gap distributions, leptokurtik shocks, non-stationary environments.

The approximate recursive general equilibrium model

Demand block

$$y_t = M^{\mathrm{HH}}\mathbb{E}_t\{y_{t+1}\} - \gamma^{-1}\big(i_t - \mathbb{E}_t\{\pi_{t+1}\} + \ln\beta\big) + b_t - \mathbb{E}_t(b_{t+1}) \qquad \text{(Aggregate demand)}$$

$$i_t = \phi_\pi\,\pi_t + \nu_t \qquad \text{(Taylor rule)}$$

Pricing block

Endogenous marginal cost

$$x_t = p_t^\star - p_{t-1} \qquad \text{(Price gap)}$$

$$p_t = \pi_t + p_{t-1} \qquad \text{(Price level)}$$

$$p_t^o = (1-\Omega)\big[\mu + (\gamma + \varphi + \nu(1+\varphi))\,y_t + u_t\big] + p_t \qquad \text{(Static target)}$$

$$\pi_t = \pi_{ss} + \beta_1\,x_t + \beta_2\,x_t^2 + \beta_3\,x_t^3 \qquad \text{(Inflation)}$$

$$h_t = \alpha_0 + \alpha_1\,x_t + \alpha_2\,x_t^2 \qquad \text{(Frequency)}$$

$$\Lambda_t\,p_t^\star = p_t^o + \beta\,\mathbb{E}_t\{(1-h_{t+1})\,\Lambda_{t+1}\,p_{t+1}^\star\} \qquad \text{(Reset price)}$$

$$\Lambda_t = 1 + \beta\,\mathbb{E}_t\{(1-h_{t+1})\,\Lambda_{t+1}\} \qquad \text{(Discounting)}$$

Cost-push

$$u_t = \rho_u\,u_{t-1} + \varrho_t$$

Demand

$$b_t = \rho_b\,b_{t-1} + \zeta_t$$

Money

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Forward looking

Cost-push	Demand	Money
$u_t = \rho_u\, u_{t-1} + \varrho_t$	$b_t = \rho_b\, b_{t-1} + \zeta_t$	$\nu_t = \rho_\nu\, \nu_{t-1} + \varsigma_t$

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Trend inflation

$$\Lambda_t\,p_t^\star = p_t^o + \beta\,\mathbb{E}_t\{(1-h_{t+1})\,\Lambda_{t+1}\,p_{t+1}^\star\} \qquad \text{(Reset price)}$$

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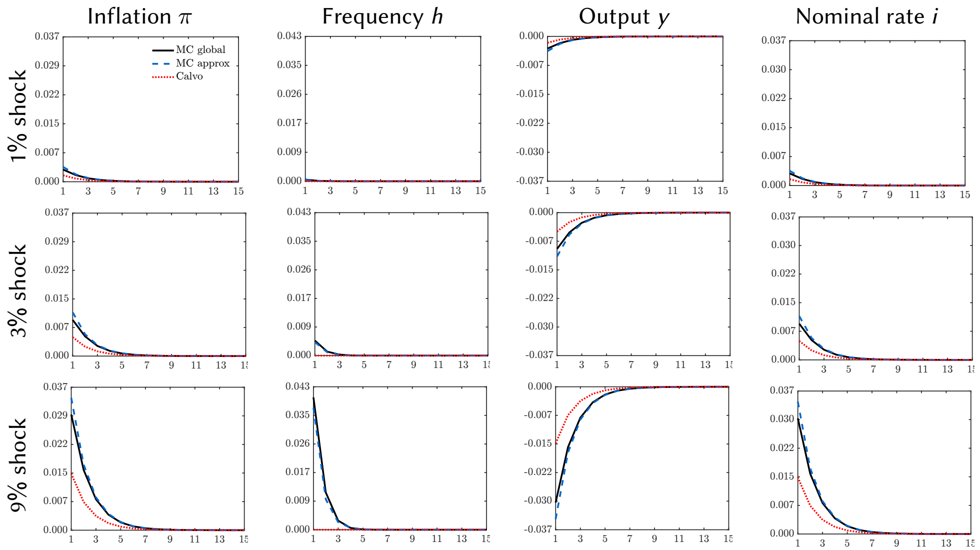
Quantitative Exercises

IRFs to cost-push shocks of varying sizes



Estimation procedure

Data (quarterly, year-over-year, 2000Q1–2025Q1, demeaned):

- Inflation — US CPI, EA HICP
- Real output growth — US and EA real GDP
- Policy rate — US Wu–Xia, EA Krippner shadow short rate

Deep parameters: $(\alpha_0, \alpha_1, \alpha_2)$ frequency, $(\beta_1, \beta_2, \beta_3)$ Phillips curve, shock parameters.

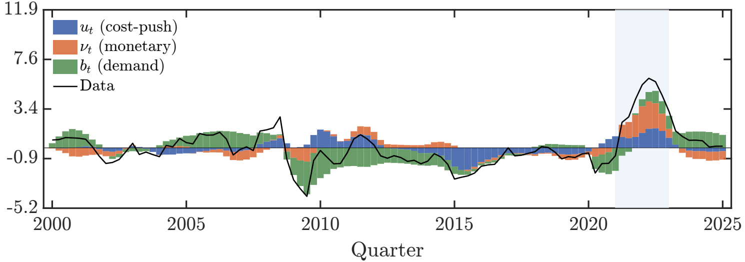
Two-stage strategy (Fernández-Villaverde & Rubio-Ramírez, 2007; Aruoba, Bocola & Schorfheide, 2017)

- **Stage 1 — order-1 Kalman.** Estimate the first-order coefficients $(\alpha_0, \beta_1, \alpha_1)$ and the shock parameters (persistences, variances, measurement errors).
- **Stage 2 — order-3 particle filter.** At a third-order solution, pin (α_2, β_3) to inflation surge (19-23).
- **Smoothing.** A backward pass recovers the full history of structural shocks.

Estimated parameters

Historical decomposition of inflation: US and Euro area

(a) United States



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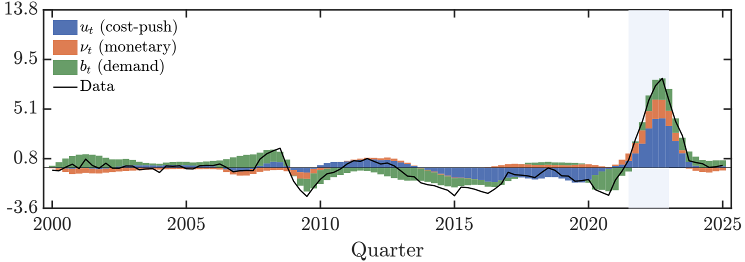
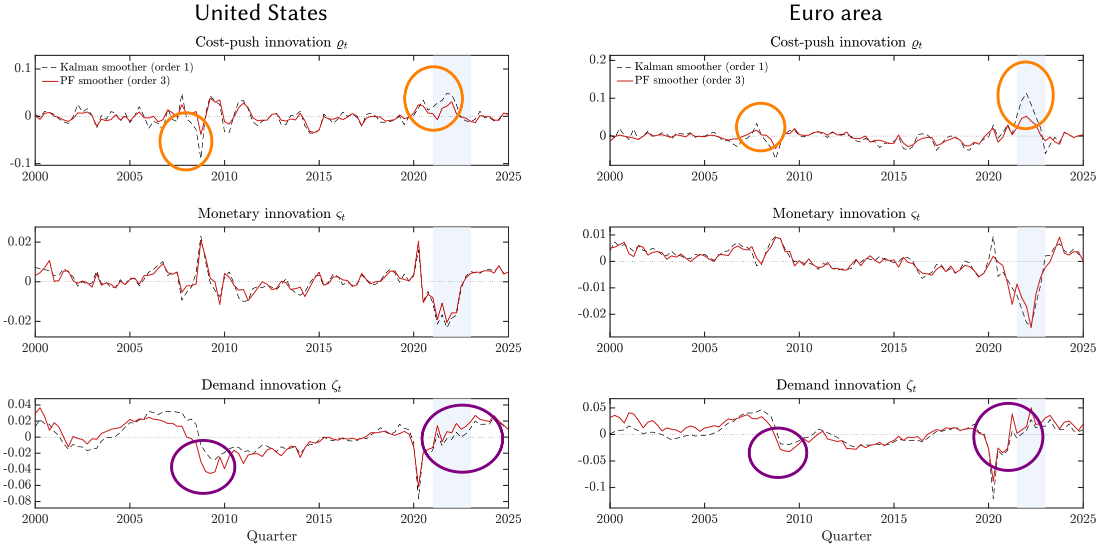
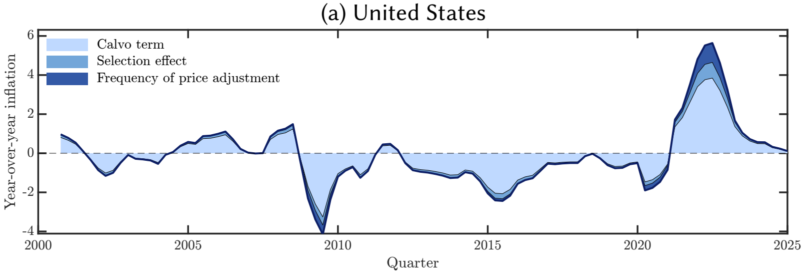
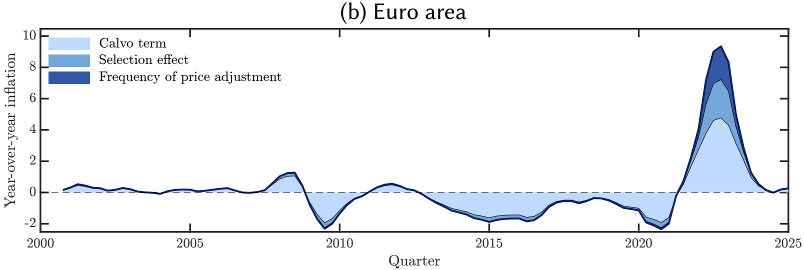


Structural shocks: order-1 vs. order-3 smoother



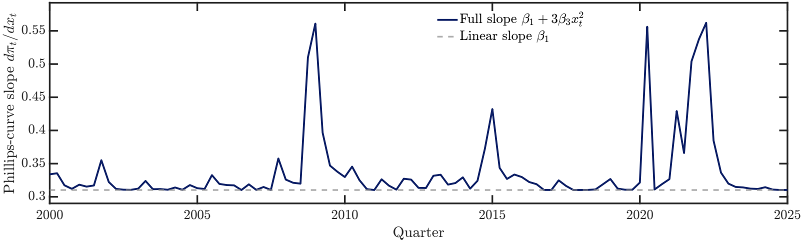
Decomposition of model-implied inflation during the surge



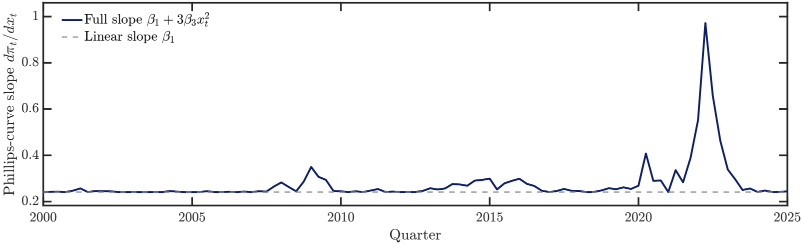


The Phillips curve's effective slope

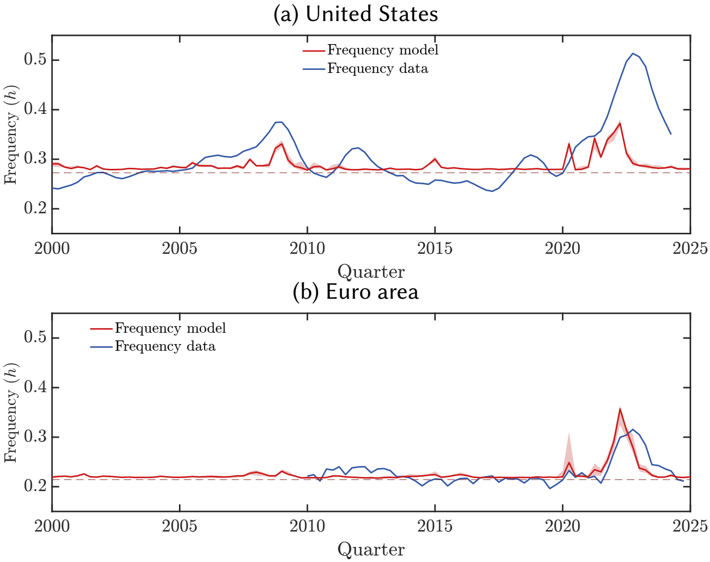
(a) United States



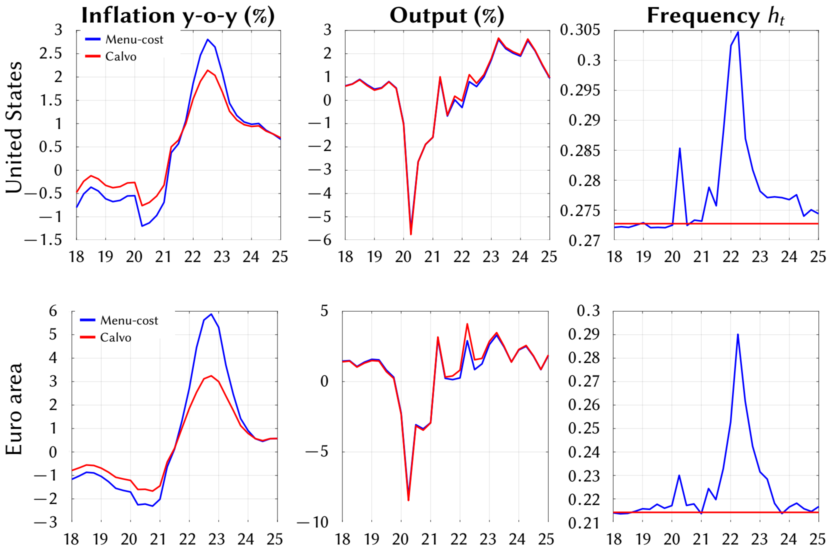
(b) Euro area



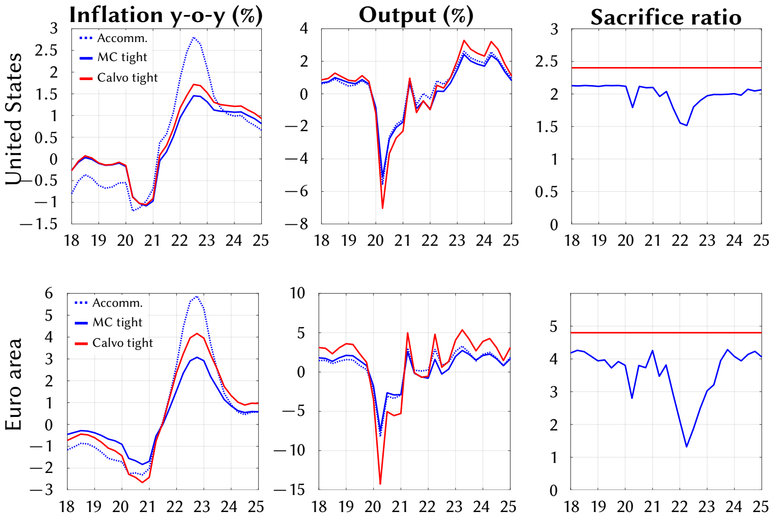
Model-implied frequency of price adjustment vs. micro data



The inflation accelerator



The sacrifice ratio (Accommodation $\phi_\pi = 1.01 \longrightarrow$ Tight $\phi_\pi = 2$)



Relative sacrifice ratio

$$\mathrm{RSR}_t = \frac{y_t^{acc} - y_t^{tight}}{\pi_t^{acc} - \pi_t^{tight}}$$

Output forgone per point of inflation reduced when the central bank switches from accommodation to tightening.

Estimated parameters: US and Euro area

		US	EA
Calibration			
Risk aversion	γ	1.00	1.00
Inv. Frisch elast.	φ	2.00	2.00
Discount factor	β	0.99	0.99
Taylor infl. coef.	ϕ_π	1.50	1.50
HH cognitive disc.	M^{HH}	0.80	0.80
Trend infl. (q)	π_{ss}	0.64%	0.54%
Strategic compl.	Ω	0.60	0.60
Quad. PC coef.	β_2	0.613	0.407
Estimated parameters			
SS adj. frequency	α_0	0.273	0.214
Lin. hazard slope	α_1	0.311	0.220
Hazard curvature	α_2	30.0	18.0
Linear PC slope	β_1	0.310	0.241
Cubic PC coef.	β_3	34.0	23.0

		US	EA
Estimated parameters (shocks)			
Cost-push SD	σ_u	0.045	0.054
Monetary SD	σ_ν	0.016	0.016
Demand SD	σ_b	0.026	0.029
Cost-push persist.	ρ_u	0.689	0.688
Monetary persist.	ρ_ν	0.301	0.301
Demand persist.	ρ_b	0.101	0.101
Infl. meas. err.	$\sigma_{e\pi}$	0.50	0.49
Output meas. err.	σ_{ey}	1.00	1.00
Rate meas. err.	σ_{ei}	0.50	0.50
Primitive parameters			
SS reset prob.	h_0	0.254	0.201
Menu-cost curv.	ϕ	9.86	7.93
Idiosyncratic var.	σ_ϵ^2	6.8×10^{-4}	4.8×10^{-4}
Diagnostics			
Kalman logdens. (o1)		−625.0	−627.2
PF loglik. (o3)		−1005.7	−1014.3
PF eff. sample		16.0%	16.0%

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Approximation of variance

Setup. $Y = X + \delta + \epsilon,\ \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2),\ \epsilon \perp X.$ Adjust $\to 0$, else keep Y; hazard $h(y) = h_0 + \phi\, y^2$.

$$\begin{array}{c}\textbf{Pre-adjustment}\\ Z = X + \epsilon \\ \text{moments } M_k = \mathbb{E}[Z^k]\end{array} \underset{\text{no aggregate shock } (\delta = 0):\ X \text{ reproduces itself } \Rightarrow\ \mathbb{E}[X^k] = m_k(0),\ M_2 = \sigma_0^2 + \sigma_\epsilon^2}{\overset{\textbf{selection}\ \ m_k(\delta) = (1 - h_0)\mathbb{E}[Y^k] - \phi\, \mathbb{E}[Y^{k+2}]}{\rightleftarrows}} \begin{array}{c}\textbf{Post-adjustment}\\ X_\delta = (1 - \mathbb{I})\, Y \\ \text{moments } m_k(\delta)\end{array}$$

Response to a shock: $x(\delta) = (1 - h_0 - 3\phi M_2)\,\delta + O(\delta^3)$; $m_2(\delta) = \sigma_0^2 + (1 - h_0 - 6\phi M_2)\,\delta^2$

$$\boxed{\sigma_t^2 = \sigma_0^2 + \sigma_{xx}^2\, x_t^2, \quad \sigma_0^2 \simeq \frac{h_0(1 - h_0)\,\sigma_\epsilon^2}{h_0^2 + 6\phi\,\sigma_\epsilon^2}, \quad \sigma_{xx}^2 = \frac{h_0(1 - h_0) - 6h_0\phi M_2 - 9\phi^2 M_2^2}{(1 - h_0 - 3\phi M_2)^2}}$$

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Closed-form coefficients: $\alpha_0, \alpha_2, \beta_1, \beta_3$

Theorem coefficients, from the variance response:

$$\alpha_0 = h_0 + \phi\,\sigma_0^2, \qquad \beta_1 = h_0 + 3\phi\,\sigma_0^2, \qquad \alpha_2 = \phi\big(1 + \sigma_{xx}^2\big), \qquad \beta_3 = \phi\big(1 + 3\sigma_{xx}^2\big).$$

Substitute $\sigma_0^2, \sigma_{xx}^2$ (previous slide) $\Rightarrow$ all in $(h_0, \phi, \sigma_\epsilon^2)$:

$$\boxed{\alpha_0 \simeq h_0 + \phi\,\frac{h_0(1-h_0)\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}, \qquad \beta_1 \simeq h_0 + 3\phi\,\frac{h_0(1-h_0)\sigma_\epsilon^2}{h_0^2 + 6\phi\sigma_\epsilon^2}, \qquad \frac{\alpha_2}{\phi} \simeq 1 + \sigma_{xx}^2, \quad \frac{\beta_3}{\phi} \simeq 1 + 3\sigma_{xx}^2}$$

Takeaway. $(h_0, \phi, \sigma_\epsilon^2)$ are pinned by frequency + dispersion of price changes — the nonlinear Phillips curve is calibrated with no global solution.

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